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**SCREENED-COULOMB ANSATZ FOR THE NON-FACTORIZABLE
RADIATIVE CORRECTIONS TO THE OFF-SHELL W^+W^- PRODUCTION****A.P. Chapovsky^{†) #)}**

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ABSTRACT

We demonstrate that the results of the complete first order calculation of the non-factorizable QED corrections to the single-inclusive cross-sections for $e^+e^- \rightarrow W^+W^- \rightarrow 4$ fermions could be very well reproduced by a simple physically motivated ansatz. The latter allows to take into account effectively the screening role of the non-Coulomb radiative mechanisms by introducing a dampening factor in front of the width-dependent part of the known first-order Coulomb correction, the so-called screened-Coulomb ansatz.

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1 Introduction

A precision study of the W -boson physics is one of the main objectives of the LEP2 programme. New unique possibilities will be opened by a future high-energy electron (muon) collider. This physics goal requires very accurate theoretical knowledge of the Standard Model predictions for the process

$$e^+e^- \rightarrow W^+W^- \rightarrow 4 \text{ fermions.} \quad (1)$$

In particular, the role of the QED radiative corrections as well as that of the finite-width effects should be understood in detail [1].

It is well known that the instability of the W bosons (the W -boson width $\Gamma_W \approx 2.1 \text{ GeV}$) can strongly modify the “stable W ” results. Special attention should be paid to the radiative interferences (both virtual and real) which interconnect the production and the decay stages of the process (1). In particular, there is a class of contributions corresponding to the so-called “charged-particle poles” [1, 2, 3, 4], which may induce strong dependence of differential distributions on the W -boson virtualities. The final-state interactions may result in non-factorizable QED radiative corrections to the Born cross-section of (1). Recall, that the level of suppression of the width-induced effects depends on the “degree of inclusiveness” of the distribution. Thus, for the totally inclusive cross-section the QED non-factorizable corrections cancel up to the terms of $\mathcal{O}(\alpha\Gamma_W/M_W)$ [2, 3]. In contrast, differential distributions could be distorted on the level of $\mathcal{O}(\alpha)$. Particular attention should be paid to the threshold region,

$$E = \sqrt{s} - 2M_W \sim \mathcal{O}(\Gamma_W). \quad (2)$$

Here the instability-induced modification of the Coulomb interaction between the slowly moving W bosons is especially significant (see for details [2, 5]). In [5] it is shown that the W -boson width effects drastically change the on-shell value of the Coulomb correction even at $E \gg \Gamma_W$, but after integration over the invariant masses of the W bosons the “stable W ” result is recovered far above the threshold region.

Recall, that in the threshold region the Coulomb contribution can be uniquely separated from the other electroweak corrections. On the contrary, in the relativistic region it is neither uniquely defined nor gauge invariant. At larger W -boson velocities β , the width-induced modifications of the differential distributions caused by other radiative mechanisms (for example, intermediate-final or final-final state interferences [2, 4]) may become just as important. These mechanisms may contribute to both factorizable and non-factorizable corrections. It is discussed in [4] that in the relativistic region a cancellation between the different sources of instability takes place. As a result, the non-factorizable corrections may vanish and the stable W result may be recovered. In the ultra-relativistic limit, $(1 - \beta) \ll 1$, such a cancellation appears quite naturally. It has, in fact, its origin in the conservation of “charged” currents (see e.g. [6, 7, 8]).

In the intermediate region, $\beta \lesssim 1$, which is relevant for the current LEP2 energy range, an analysis of the non-factorizable corrections to the differential cross-sections of W^+W^- production requires detailed studies.

During the last few years there has been a significant progress in our understanding of the radiative effects in the off-shell gauge-boson pair production [1]. In particular, recently a complete calculation of the non-factorizable corrections to the process (1) has been independently performed by two groups [9, 10, 11] (see also [4]). The results are in a very good agreement with each other.

In [8] an attempt has been made to estimate the possible screening impact of other radiative interference mechanisms on the Coulomb scenario in the relativistic regime. Driven by physical intuition, the consequences of the so-called dampened- (screened-) Coulomb ansatz have been briefly considered. Within this ansatz an extra screening factor $(1 - \beta)^2$ is introduced in front of the width-dependent arctan term in the known first order unstable Coulomb formula (see e.g. [5, 12]). Such a simple prescription was motivated by a model analysis of [4], where a simple scenario was considered in which one of the W bosons was assumed to be stable. Another example of the cancellation of the off-shell effects at relativistic energies has been known for quite a while (see e.g. [6, 13]). When considering the gluon radiation corresponding to the top production and decay at very high energies, one observes that the width-dependent effects vanish when the emission at the production and decay stages are added coherently.

An obvious attractiveness of the screened-Coulomb ansatz is that it is very simple and readily allows for a transparent physical interpretation. It could be useful as well from the point of view of practical applications. It requires, however, a special detailed study in order to understand whether this scenario can be taken as a realistic plausible baseline.

It is the aim of this paper to perform a detailed comparison of the screened-Coulomb ansatz with the results of the recent calculations [9, 10, 11]. It appears that this simple prescription provides one with a surprisingly reasonable quantitative understanding of the screening of the non-factorizable terms at higher energies. The results of this paper can equally well be applied to the $\gamma\gamma$ initiated processes.

The paper is organized as follows. In Section 2 some basic formulae are presented. In Section 3 we study numerically several characteristic observables. We conclude in Section 4. Appendix contains a quantitative analysis of the screening effects basing on the explicit Feynman diagram calculations.

2 Ansatz for the non-factorizable corrections

In the Born approximation for the process (1) there are three (signal) diagrams where two resonant W bosons are produced and the background diagrams where, at most, one resonant W -boson is formed. The background diagrams are typically suppressed by $\mathcal{O}(\Gamma_W/M_W)$ [$\mathcal{O}(\Gamma_W^2/M_W^2)$] with respect to the leading double resonant contributions.

The currently most favourable approach adopted for the calculation of the radiative corrections to the processes involving unstable particles is the so-called *pole-scheme* [14]. In the double-pole approximation one considers the complete off-shell process as a superposition of

the production of a pair of unstable particles and their subsequent decays. The radiative effects are then naturally separated into two groups: factorizable and non-factorizable. The first type includes radiative corrections which can be unambiguously attributed either to the production or to the decay stage of the process. They exhibit simple analytical behaviour in the limit $\Gamma_W \rightarrow 0$. The second type corresponds to the radiative interconnections between various stages of the process.

It is instructive to trace the physical origin of such separation not too far from the threshold. When considering soft photons, $k^0 = \omega \ll M_W$, the production and decay of the W bosons can be regarded essentially as point-like processes with a characteristic time scale $t_{\text{char}} \sim 1/M_W$. However, due to the W -decays, various stages are separated in time by an intervals $\tau \sim 1/\Gamma_W$. When we average over the times between W -pair production and the W -decays a significant interconnection occurs only in the $\omega \lesssim \Gamma_W$ domain. This results in the non-factorizable correction. The contribution to these corrections caused by the hard photons is power suppressed (see e.g. [2, 13, 15]).

When examining the process (1) one distinguishes three energy domains:

- Threshold region (2) where W 's are moving with a small velocity with respect to each other, $\beta \sim \sqrt{\Gamma_W/M_W} \ll 1$.
- Non-relativistic region, $\Gamma_W \ll E \ll M_W$, where the velocity of the W 's is still a small parameter, $\beta \ll 1$, but the centre-of-mass energy is sufficiently far from threshold.
- Relativistic region, $E \sim M_W$, where velocity of the W 's is not a small parameter any more, $\beta \sim 1$.

Recall, that in the threshold and in the non-relativistic region the main contribution to the radiative corrections comes from the Coulomb interaction (see [5, 12]). All other effects are suppressed by $\mathcal{O}(\beta)$. Near threshold the Coulomb contribution dominates the instability effects. In the relativistic region the terms suppressed in the non-relativistic region are not small and should be taken into account. The explicit calculation of the complete non-factorizable correction performed in [9] uses the “far from threshold” (FFT) approximation, which assumes that $\Gamma_W \ll E$. The accuracy of this approximation is $\mathcal{O}(\Gamma_W/E)$. This approximation breaks down in the threshold region, but it is valid in the non-relativistic region (far from threshold) and in the relativistic region. Note that in the non-relativistic region the calculation of the complete non-factorizable correction agrees with the calculation of the off-shell Coulomb effect within the adopted approximations.

We discuss below a simple ansatz based of the Coulomb result (screened-Coulomb) which appears to be in a good agreement with the complete calculation of the non-factorizable corrections in the relativistic as well as in the non-relativistic region. Of course, one cannot expect that a simple unique prescription exists, which would allow to reproduce reasonably well the results of the explicit complete calculations of the non-factorizable corrections to the

arbitrary differential distribution.¹ Below we concentrate on the quantities, which are inclusive with respect to all the decay and production angles, such as the invariant mass spectrum of a W boson.

Since the calculation of the non-factorizable corrections in [9, 10, 11] had been performed in the FFT approximation, we shall remain within the same scheme for the screened-Coulomb ansatz. This means that we shall not consider the threshold region here. The reader is reminded that the latter region has been studied in detail elsewhere (see e.g. [5, 12]). For the reference purposes we consider also a model case when the cross-section is corrected by the Coulomb effect only. Then the differential distribution over an observable X can be written in the following form²

$$\frac{d\sigma_{\text{“Coul”}}}{dX} = \frac{d\sigma_{\text{Born}}}{dX} \left(1 + \delta_{\text{“Coul”}} \right), \quad \delta_{\text{“Coul”}} = \delta_{\text{“Coul”}}^{\text{on-shell}} + \delta_{\text{“Coul”}}^{\text{nf}} \quad (3)$$

$$\delta_{\text{“Coul”}}^{\text{on-shell}} = \frac{\alpha\pi}{2\beta}, \quad \delta_{\text{“Coul”}}^{\text{nf}} = -\frac{\alpha}{\beta} \arctan\left(\frac{M_1^2 + M_2^2 - 2M_W^2}{2M_W\Gamma_W}\right), \quad (4)$$

where M_1 and M_2 are the invariant masses of the W bosons, β is their on-shell velocity

$$\beta = \sqrt{1 - 4M_W^2/s}, \quad (5)$$

and $d\sigma_{\text{Born}}/dX$ is the on-shell Born cross-section. This is the leading contribution to the radiative correction in the non-relativistic region. All other contributions, which were neglected, are suppressed by, at least, $\mathcal{O}(\beta, \Gamma_W/E)$. Let us emphasise that outside of the threshold region the Coulomb approach is just an oversimplified extreme and, naturally, is not supposed to correspond to the true physics. Note, that throughout this paper the so-called fixed-width scheme is used, where Γ_W is the on-shell W -boson width.

The two terms in (4) have different nature. The first one represents the factorizable part of the Coulomb interaction. It is completely the same as the familiar Coulomb effect for the stable case. It should be noted again that at high energies, where β is not a small parameter, this correction is of the same order as the rest of the radiative corrections, and is not enhanced in any way. Typically, the leading contribution coming from radiative corrections goes from $\sim \alpha\pi/\beta$ at threshold to $\sim \alpha/\pi$ far from threshold.

The second term is the non-factorizable part of the Coulomb correction. It arises due to the instability effects. It averages to zero when integrated over the invariant masses. As discussed in [2, 3] (see also [5]), this is a general feature of the non-factorizable corrections.

The physical reason for the separation between the factorizable and non-factorizable corrections is rooted in the difference in the characteristic energies and momenta of the photons responsible for the different terms in (4).

¹When considering the angular distributions of the final-state fermions the general arguments based on the conservation of the “charged currents” (see e.g. [6]) may be not applicable, and the screened-Coulomb ansatz could be irrelevant. The largest discrepancies should be observed near the edges of the kinematic phase space where the corrections are the largest, but the event statistics is very limited. A detailed study of the dependence of the non-factorizable corrections on the fermion angles has been presented in [9, 10, 11].

²We are considering here and in what follows only the first-order Coulomb formulae. As shown in [8, 16] the higher order Coulomb effects are practically negligible.

In order to gain insight let us consider the diagram with the photon exchange between the two W bosons. The denominator of the propagator of the W boson with the 4-momentum p_1^μ is

$$k^2 + 2kp_1 + D_1, \quad D_1 = p_1^2 - M_W^2 + i\Gamma_W M_W. \quad (6)$$

Not too far from threshold for the on-shell (factorizable) part of the Coulomb effect photons with energies $\omega \sim \beta^2 M_W$ and momenta $|\vec{k}| \sim \beta M_W$ are essential. It is worth-while to recall that $1/(\beta^2 M_W)$ is the typical interaction time between the W bosons, see [5]. In such a case k^2 can not be neglected in the W -boson propagator, contrary to the $\Gamma_W M_W$ term, see [2, 5]. Therefore, the Coulomb effect here remains unchanged by the instability of the W bosons. On the other hand, only the photons with the energies $\omega \sim \Gamma_W$ and momenta $|\vec{k}| \sim \Gamma_W/\beta$ give the leading contribution to the off-shell part of the Coulomb effect. Note that β/Γ_W is the typical spatial separation between the diverging W bosons [12]. Far from threshold, at $M_W \gg E \gg \Gamma_W$, the two regions in the photon energy-momentum space are well separated. Because of this fact the effects are additive. Near threshold, where $E \sim \Gamma_W$, the two regions start to overlap, which is precisely the reason why our approach to the calculation of the double pole residues becomes invalid.

As has been already mentioned, in the relativistic domain Coulomb correction does not account correctly for all the effects. Instead the complete non-factorizable corrections are required

$$\frac{d\sigma_{\text{nf}}}{dX} = \frac{d\sigma_{\text{Born}}}{dX} (1 + \delta_{\text{nf}}). \quad (7)$$

The explicit expressions for δ_{nf} [9, 10, 11] are rather lengthy, and for the purposes of this paper there is no need to present them here.

Motivated by [4, 8] we would like to check whether the complete non-factorizable corrections could be approximated reasonably well by a simple ansatz based on the screening of the non-factorizable (off-shell) part of the Coulomb effect

$$\frac{d\sigma_{\text{Ans}}}{dX} = \frac{d\sigma_{\text{Born}}}{dX} (1 + \delta_{\text{Ans}}), \quad (8)$$

where

$$\delta_{\text{Ans}} = \delta_{\text{Coul}}^{\text{nf}} (1 - \beta)^2. \quad (9)$$

Non-factorizable corrections distort the Breit-Wigner distribution over the invariant mass of the W -boson. This results, in particular, in the shift of the maximum of the invariant mass distribution. The potential importance of this effect is quite transparent since such a shift may affect the measurement of the mass of the W -boson. It is possible to estimate this shift from the relative non-factorizable correction to invariant mass distribution. We will consider specifically the distribution over the average invariant mass $\bar{M} = (M_1 + M_2)/2$. The standard expression for the linearized shift is

$$\Delta \bar{M} = \frac{1}{8} \Gamma_W^2 \frac{d\delta_{\text{nf}}(\bar{M})}{d\bar{M}} \Big|_{\bar{M}=M_W}. \quad (10)$$

Basing on the ansatz prescription (8) and (9) for the non-factorizable correction one arrives at the very simple formula for the shift (see also Ref. [4])

$$\Delta\bar{M} = -\frac{\alpha(1-\beta)^2}{4\beta}\Gamma_W. \quad (11)$$

In the following Section we shall investigate numerically how this ansatz approximates the complete non-factorizable correction to the single-inclusive distributions at various energies. In all cases a very good agreement is established. We show some specific examples, which illustrate this statement.

3 Numerical results

In the following calculations we assume

$$\alpha = 1/137.0359895, \quad \alpha(M_Z) = 1/127.9, \quad \sin^2\theta_W = 0.223,$$

$$M_W = 80.41 \text{ GeV}, \quad \Gamma_W = 2.06 \text{ GeV}, \quad M_Z = 91.187 \text{ GeV}, \quad \Gamma_Z = 2.49 \text{ GeV}. \quad (12)$$

We use $\alpha(M_Z)$ to calculate the Born cross-sections, and α to calculate the radiative corrections. Results are presented in the LEP2 energy range $\sqrt{s} = 160 - 200 \text{ GeV}$ and for three discrete energies: $\sqrt{s} = 172, 183$ and 195 GeV . Several comparisons are presented between the results of the complete calculation of the non-factorizable corrections, [9], the expectations based on the screened-Coulomb ansatz, and the model unscreened-Coulomb prescription. The latter scenario can help one to assess the impact of the non-Coulomb radiative interferences on the width-dependent effects. It should be stressed that throughout the paper the non-factorizable corrections are calculated for the purely leptonic final state (for example, $\mu^+\nu_\mu e^-\bar{\nu}_e$). Strictly speaking, outside threshold region non-factorizable corrections to other final states (i.e. semi-hadronic and purely hadronic final states) are not identical [11], but the differences are, in fact, not so large.

Fig. 1 compares the distribution over the average invariant mass $\bar{M} = (M_1 + M_2)/2$ in the three scenarios above at $\sqrt{s} = 172, 183$ and 195 GeV . Fig. 2 shows the additional mass shift $\Delta\bar{M}$ due to the non-factorizable effects as a function of the collider energy. The expectation corresponding to Eq. (11) is also shown. One observes a remarkable agreement between the result of the complete calculations and a simple screening recipe (11) for the mass shift.

For practical purposes it is useful to analyse the impact of the instability effects on W -momentum distribution, see e.g. [8]. Fig. 3 compares the results for the differential momentum distribution $d\sigma/dp$ in the three scenarios at $\sqrt{s} = 172, 183$ and 195 GeV .

Figs. 1 and 3 clearly show the dampening role of the screening factor $(1-\beta)^2$. In particular, a sharp increase in δ_{nf} around $p = p_0 = \sqrt{E\bar{M}_W}$ becomes much less pronounced as compared to the unscreened case, see also [8]. The plots demonstrate that the screened-Coulomb ansatz is quite reliable even for momenta that significantly deviate from p_0 .

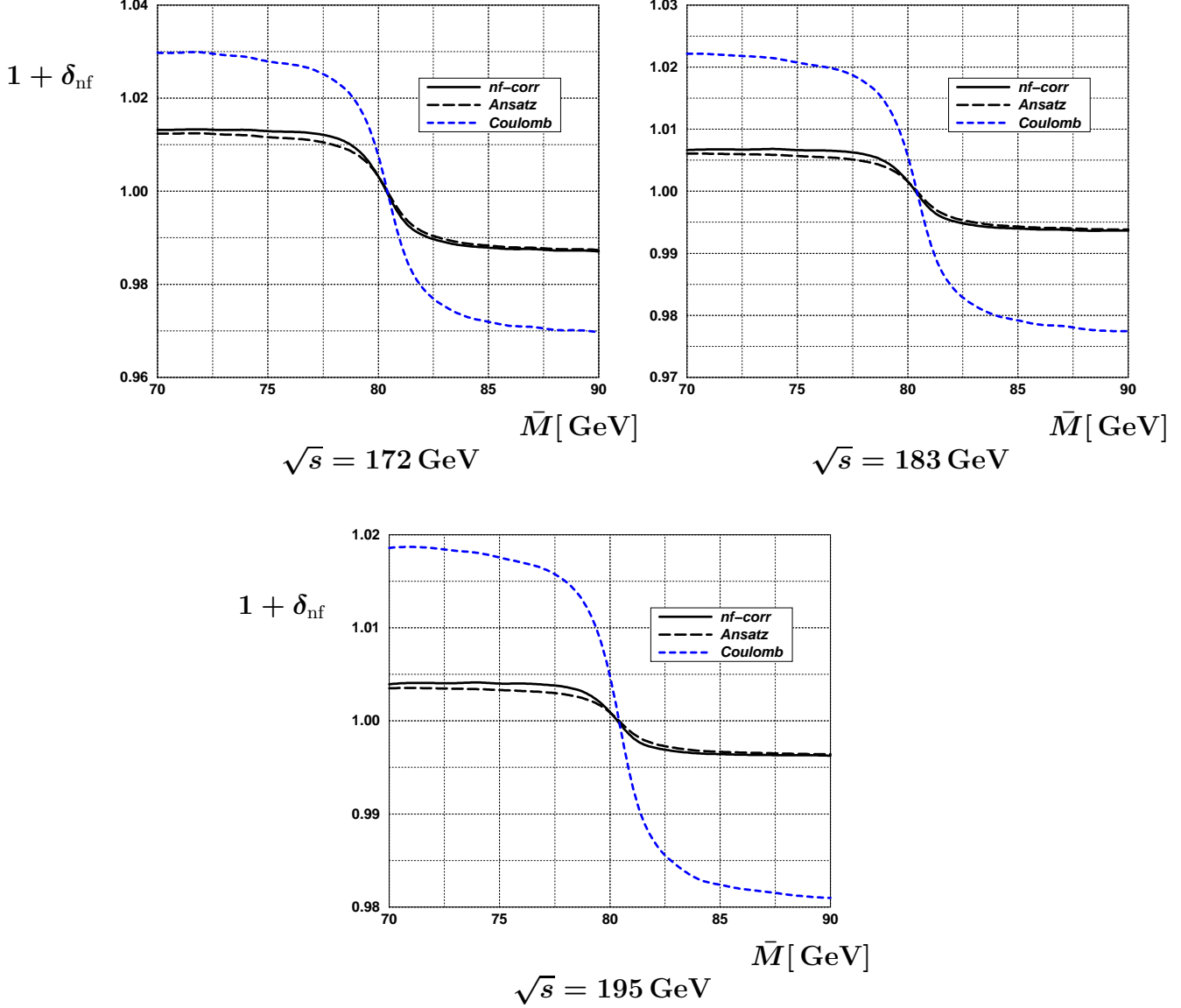


Figure 1: The complete non-factorizable correction to the distribution over the average invariant mass $\bar{M} = (M_1 + M_2)/2$ at $\sqrt{s} = 172, 183$ and 195 GeV , as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios.

Finally, recall that the high energy behaviour of the non-factorizable corrections to the ZZ production is of a special interest, in particular, because of a certain resemblance between these QED interference effects and colour interconnection phenomena in the gauge boson pair production, see, e.g. [17]. An explicit numerical calculation confirms the presence of the same screening $(1 - \beta)^2$ factor in this case, see Figs. 4 and 5. Analogously to the WW case, it also has its origin in the conservation of currents.

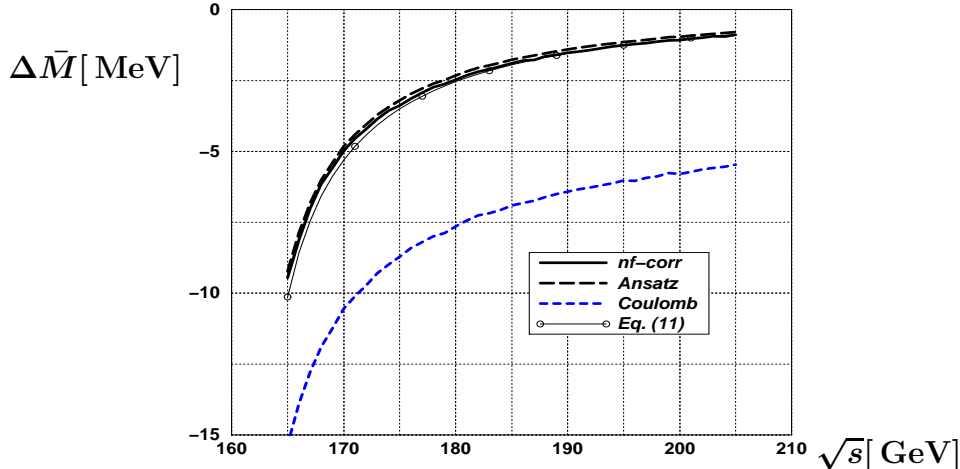


Figure 2: The additional shift of the maximum in the W average invariant mass distribution due to the non-factorizable correction as a function of the collider energy, as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios.

4 Conclusions

The success of the precision studies of the W boson physics relies on an accurate theoretical knowledge of the details of the production and decay mechanisms. The instability of the W bosons can, in principle, strongly modify the standard “stable W ” results. An important role can be played by the radiative interference effects, which prevent the final state in $e^+e^- \rightarrow W^+W^- \rightarrow 4$ fermions from being treated as two separate W decays. Thus, purely QED interaction between two unstable W bosons induces non-factorizable corrections to various final state distributions. The complete analytical calculations of these corrections have been performed only recently [9, 10, 11]. In this paper we demonstrate explicitly that a simple physically motivated ansatz allows one to approximate the non-factorizable corrections to the single-inclusive final state distributions with a surprisingly good accuracy. This approach makes the physical insight into the effects of instability of the W -pair production quite transparent.

One has to bear in mind that, typically, the order of magnitude of the non-factorizable corrections does not exceed one percent, and their practical relevance strongly depends on the requirements of the experiment. In particular, they could match the expected accuracy of measurements at a future lepton collider.

Finally, let us note that similar screening scenario could, in principle, provide a useful framework for studies of the QCD final-state interactions in $e^+e^- \rightarrow t\bar{t}$ and of the colour interconnection effects in the W^+W^- -production, see e.g. [17].

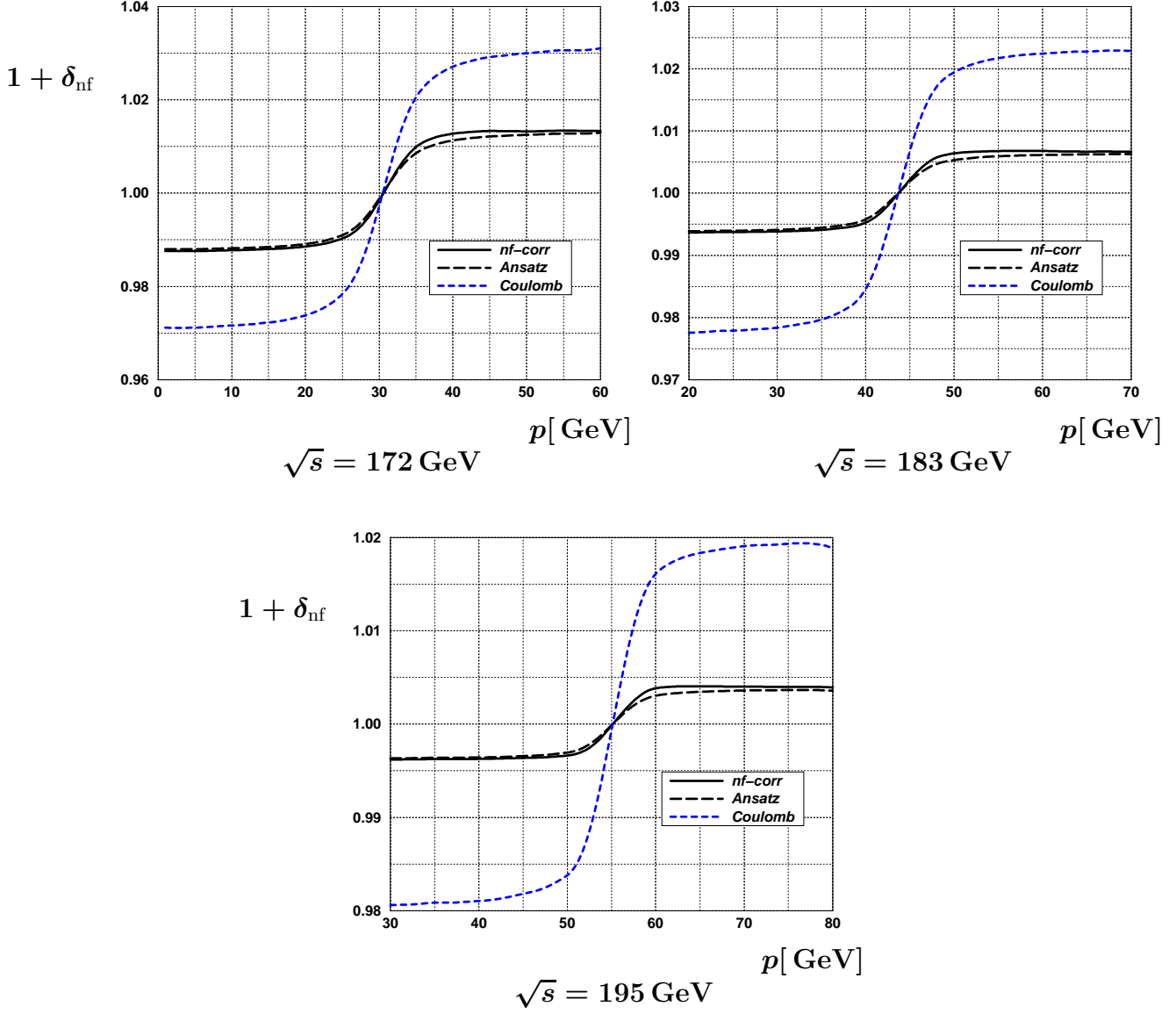


Figure 3: The complete non-factorizable correction to the distribution over the W -momentum at $\sqrt{s} = 172, 183$ and 195 GeV , as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios.

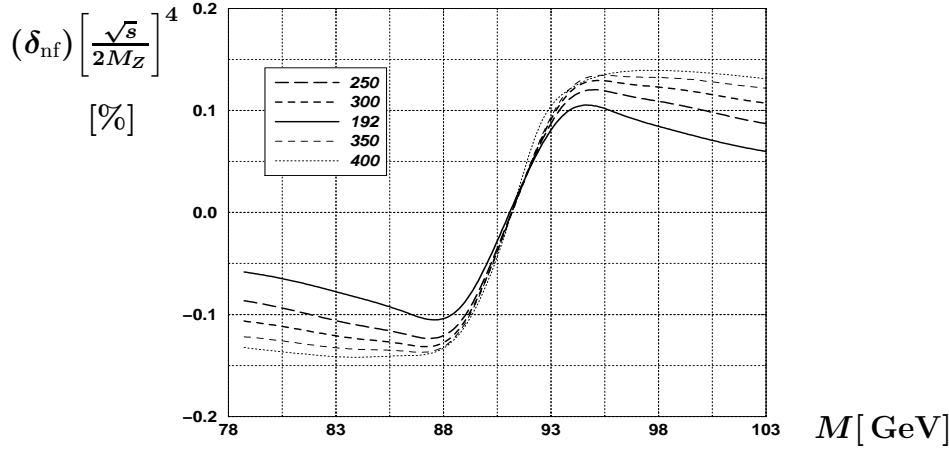


Figure 4: The non-factorizable correction to the distribution over the Z -boson invariant mass in $e^+e^- \rightarrow ZZ \rightarrow d\bar{d}u\bar{u}$. In order to elucidate the role of the screening factor, the non-factorizable correction, δ_{nf} , was multiplied by the $(\sqrt{s}/2M_Z)^4$ factor. The curves are given for $\sqrt{s} = 192, 250, 300, 350$ and 400 GeV.

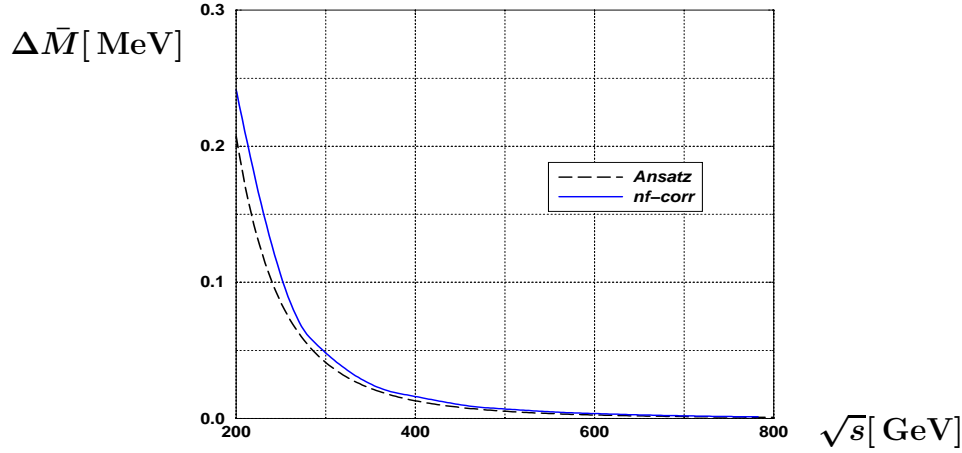


Figure 5: The additional shift of the maximum in the Z -boson invariant mass distribution due to the non-factorizable correction as a function of the collider energy, as compared to the expectations from the screened ansatz scenario: $\delta_{\text{nf}}^{ZZ} \sim (2M_Z/\sqrt{s})^4$.

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Appendix

Screening effects: quantitative discussion

The aim of this Appendix is to expose the origin of the screening $(1 - \beta)^2$ factor basing on the explicit evaluation of Feynman diagrams.

We first consider a model case [4] (see also [7]) in which one of the W bosons is assumed to be stable and the other one has the standard decay modes with decay width Γ_W . In this way we can gain insight into the analytical structure of the interference effects without encountering the complications which occur in the case of two unstable bosons. Our final result for the non-factorizable corrections in such a hypothetical case fully agrees with that in Ref. [4]. Nevertheless, we find it instructive to present our alternative (more transparent) derivation. The results obtained within this simpler model will be used in the discussion of the realistic process when both W bosons are off-shell.

In this model example the virtual non-factorizable correction can be written as the interference between two currents:

$$d\sigma_{\text{nf}} = d\sigma_{\text{Born}} 2\text{Re} 4\pi\alpha i \int \frac{d^4k}{(2\pi)^4 k^2} \frac{p_2^\mu}{(-p_2 k)} \left[\frac{p_1^\mu}{p_1 k} - \frac{k_1^\mu}{k_1 k} \right] \frac{D_1}{D_1 + 2p_1 k}, \quad (\text{A.1})$$

where

$$D_1 = p_1^2 - M_W^2 + iM_W\Gamma_W, \quad (\text{A.2})$$

p_1 and p_2 are the 4-momenta of the unstable W^- -boson and stable W^+ -boson respectively, and k_1 and k_1' are the 4-momenta of the decay products of the off-shell W -boson, $p_1 = k_1 + k_1'$.

Note, that here and in what follows the k^2 terms in the propagators of the radiating particles are neglected. Outside the threshold domain an account of these terms gives a negligibly small (order Γ_W/E) effect.

We must also include the corresponding to (A.1) contribution coming from the real photon radiative interference. Note, that throughout this paper we do not consider the non-factorizable corrections involving the initial state radiation because of the cancellation between the virtual and real pieces, see for details e.g. the first reference in [2]. This also allows one to apply the results to the photon-photon initiated process.

The integrand in (A.1) has two poles in the upper-half k^0 -plane (the photon pole and the W^- -pole). The remaining poles are located in the lower-half-plane. When we perform the dk^0 integration by closing the contour in the upper-half-plane, we see immediately that the contributions from the real and virtual pieces cancel each other. This exemplifies the well-known cancellation between the real and virtual emissions.³ Thus, the only non-zero contribution comes from the W^- -pole. We found it especially convenient to perform the analysis of this contribution in the rest frame of the W^- . In this Lorentz frame the particle 4-momenta can be written as: $p_1^\mu = (E_1; \vec{p}_1)$, $|\vec{p}_1| = \tilde{\beta}E_1$, $p_2^\mu = (M_W, \vec{0})$, $k_1^\mu = (\epsilon_1; \vec{k}_1)$

³This cancellation is similar to the case of the non-factorizable corrections caused by the initial-final state radiative interference, see [2, 4].

Let us now evaluate the integral

$$I = \text{Re } i \int \frac{d^4 k}{k^2} \text{Pole}^{\text{up}} \frac{(p_2 k_1)}{(-p_2 k)(k_2 k)} \frac{D_1}{D_1 + 2p_1 k}. \quad (\text{A.3})$$

“Pole^{up}” denotes that the residue should be taken in the poles located in the upper-half-plane, thus only the $(-p_2 k)$ -pole contributes. In the W^- rest frame this term is just $-p_2 k = -\omega M_W + i0$. Let us take the residue of this pole and use the cylindrical coordinates

$$I = -\text{Re } 2\pi \epsilon_1 \int \frac{dk_{\parallel} k_{\perp} dk_{\perp} d\phi}{k_{\parallel}^2 + k_{\perp}^2} \frac{1}{(-k_{\parallel} k_{1\parallel} - \vec{k}_{\perp} \vec{k}_{1\perp} + i0)} \frac{D_1}{D_1 - 2|\vec{p}|k_{\parallel}}. \quad (\text{A.4})$$

Now we carry out the k_{\parallel} integration. The integrand has radiating particle poles, which are located in the upper-half k_{\parallel} -plane. The photon poles occur at $k_{\parallel} = \pm i k_{\perp}$. Now we close the contour in the lower-half-plane in order to avoid all charged particle poles. Then the interference contribution becomes

$$I = -\text{Re } 2\pi^2 \epsilon_1 \int \frac{dk_{\perp} d\phi}{k_{\perp}} \frac{1}{(ik_{1\parallel} - |\vec{k}_{1\perp}| \cos \phi)} \frac{D_1}{D_1 + 2|\vec{p}|k_{\perp}}. \quad (\text{A.5})$$

The integration over the azimuthal angle ϕ is quite straightforward. The integral over k_{\perp} is infrared divergent, but the divergent piece is pure imaginary. Finally, we arrive at

$$I \sim \frac{\epsilon_1}{|\vec{k}_1|} \text{Re } i \ln \frac{D_1}{i}. \quad (\text{A.6})$$

So far we have evaluated only the second term in (A.1). The first term can be treated in an analogous way after the substitution $k_1 \rightarrow p_1$. The complete non-factorizable correction is then given by

$$\delta_{\text{nf}} \sim \left(\frac{E}{|\vec{p}_1|} - \frac{\epsilon_1}{|\vec{k}_1|} \right) \text{Re } i \ln \frac{D_1}{i} = \frac{1 - \tilde{\beta}}{\tilde{\beta}} \text{Re } i \ln \frac{D_1}{i}. \quad (\text{A.7})$$

The fact that the pre-logarithmic factor approaches zero as $\tilde{\beta} \rightarrow 1$ is a direct consequence of the charged current conservation. Recall, that $\tilde{\beta} = \sqrt{1 - M_W^2/E_1^2}$ is taken in the system where W^- is at rest. Now we return to the centre-of-mass frame, where the velocity of the W 's is β ($\beta = \sqrt{1 - 4M_W^2/s}$). It is connected to $\tilde{\beta}$ by $\tilde{\beta} = 2\beta/(1 + \beta^2)$. This allows to present the complete non-factorizable correction in the canonical ansatz form

$$\delta_{\text{nf}} \sim \frac{(1 - \beta)^2}{2\beta} \arctan \frac{M_1^2 - M_W^2}{M_W \Gamma_W}. \quad (\text{A.8})$$

Now we turn to the realistic case of two off-shell W -bosons. We shall concentrate on the high energy behaviour of the complete non-factorizable correction.

The virtual non-factorizable correction can be presented in a standard form as a sum of the current interferences

$$\mathcal{M}_{\text{nf}}^{\text{virt}} = i\mathcal{M}_{\text{Born}} \int \frac{d^4 k}{(2\pi)^4 k^2} \left[(\mathcal{J}_0 \mathcal{J}_+) + (\mathcal{J}_0 \mathcal{J}_-) + (\mathcal{J}_+ \mathcal{J}_-) \right]. \quad (\text{A.9})$$

The currents are given by

$$\mathcal{J}_0^\mu = e \left[\frac{p_1^\mu}{kp_1} + \frac{p_2^\mu}{-kp_2} \right], \quad \mathcal{J}_+^\mu = -e \left[\frac{p_1^\mu}{kp_1} - \frac{k_1^\mu}{kk_1} \right] \frac{D_1}{D_1 + 2kp_1}, \quad \mathcal{J}_-^\mu = -e \left[\frac{p_2^\mu}{-kp_2} - \frac{k_2^\mu}{-kk_2} \right] \frac{D_2}{D_2 - 2kp_2} \quad (\text{A.10})$$

Here $p_{1,2}$ are the 4- momenta of the W -bosons, and $k_{1,2}$ are the 4-momenta of the corresponding charged decay products. There is also a corresponding contribution coming from the real photon radiation interferences. The first and the second terms in (A.9) can be treated exactly in the same way as a model case before. Therefore, we shall concentrate on the third term which has a different analytical structure.

Recall, that at higher energies the dominant contribution to the radiative interference effects comes from the photons (real or virtual) with the energies

$$\omega \sim \Gamma_W \frac{M_W}{E_W}, \quad 2E_W = \sqrt{s}, \quad (\text{A.11})$$

see e.g. [13]. One can arrive at the same conclusion from an explicit estimate of the dominant contribution to the integral (A.10).

To be specific we concentrate below on the typical case of the W decay mass distribution. Therefore, it is assumed that the integration over the decay products has been already carried out. Let us analyze the consequences of this integration for the Born decay cross-section and for the non-factorizable currents (A.10). First, recall that the Born decay matrix element squared can be written as

$$\mathcal{M}_{\text{dec}}^\mu \mathcal{M}_{\text{dec}}^{*\nu} \sim \frac{1}{4} \text{Sp} \left[\gamma^\mu (1 - \gamma^5) \not{k}_1 \gamma^\nu (\not{p}_1 - \not{k}_1) \right] = \Delta_V^{\mu\nu} - i \Delta_A^{\mu\nu}. \quad (\text{A.12})$$

$$\Delta_V^{\mu\nu} = k_1^\mu p_1^\nu + k_1^\nu p_1^\mu - 2k_1^\mu k_1^\nu - g^{\mu\nu} \frac{M_W^2}{2}, \quad \Delta_A^{\mu\nu} = \epsilon^{\mu\nu k_1 p_1},$$

where indices μ and ν are to be contracted with the corresponding ones in the production part of the Born cross-section. We used the notations $\epsilon^{\mu\nu\rho p} = \epsilon^{\mu\nu\rho\alpha} p_\alpha$. Let us start from the vector piece $\Delta_V^{\mu\nu}$.

The Born decay cross-section integrated over the phase-space of the decay products is given by

$$\mathcal{I}_{\text{Born}}^{\mu\nu} = \int d^4 k_1 \delta(k_1^2) \delta(M_W^2 - 2(p_1 \cdot k_1)) \times \Delta_V^{\mu\nu} = \frac{\pi}{6} M_W^2 \left[g^{\mu\nu} - \frac{p_1^\mu p_1^\nu}{M_W^2} \right]. \quad (\text{A.13})$$

Consider now an integral over the corresponding non-factorizable current

$$\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} = \int d^4 k_1 \delta(k_1^2) \delta(M_W^2 - 2(p_1 \cdot k_1)) \times \Delta_V^{\mu\nu} \times \left[\frac{p_1^\alpha}{kp_1} - \frac{k_1^\alpha}{kk_1} \right]. \quad (\text{A.14})$$

Tensor $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha}$ can depend only on the 4-vectors p_1^μ and k^μ and has the following general features:

- $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} = \mathcal{I}_{\text{nf}, V}^{\nu\mu\alpha}$,
 - $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} p_{1\mu} = 0$,
 - $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} k_\alpha = 0$,
 - $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} g_{\mu\alpha} = 0$.
- (A.15)

It is convenient to carry out the integration in the W -boson rest frame. The integral $\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha}$ simplifies in the high energy limit, if one recalls that only the soft photons (A.11) are responsible for the non-factorizable correction. Then $p_1^\mu \sim E_W$, and $k^\mu \sim \Gamma_W M_W / E_W$.

$$\mathcal{I}_{\text{nf}, V}^{\mu\nu\alpha} = A \left(\frac{p_1^\alpha}{k p_1} - \frac{k^\alpha}{k^2} \right) \left[g^{\mu\nu} + \frac{k^2}{(k p_1)^2} p_1^\mu p_1^\nu + M_W^2 \frac{k^\mu k^\nu}{(k p_1)^2} - \frac{1}{(k p_1)} \left[k^\mu p_1^\nu + p_1^\mu k^\nu \right] \right], \quad (\text{A.16})$$

$$A = \frac{\pi}{4} \frac{M_W^4 k^2}{(k p_1)^2} \left[1 + \frac{1}{2} \ln \frac{M_W^2 k^2}{4(k p_1)^2} \right]. \quad (\text{A.17})$$

Note that $A \sim E_W^{-2}$ (we keep track of the energy dependence only), since $(k p_1) \sim M_W \Gamma_W$ and $k^2 \sim \Gamma_W^2 M_W^2 / E_W^2$. Recall also that the $\mu\nu$ tensor in the equation above is of the same order (or less) as in the Born approximation, see (A.13).

Now we can readily obtain an upper limit for the third term in the square brackets in the integrand in (A.9).

$$\begin{aligned} \mathcal{M}_{\text{nf}, V}^{\text{virt}} &\sim \int d^4 k_1 \delta(k_1^2) \delta(M_W^2 - 2(p_1 \cdot k_1)) \times \Delta_V^{\mu\nu}(p_1, k_1) \times \\ &\times \int d^4 k_2 \delta(k_2^2) \delta(M_W^2 - 2(p_2 \cdot k_2)) \times \Delta_V^{\mu'\nu'}(p_2, k_2) \times \int \frac{d^4 k}{(2\pi)^4 k^2} (\mathcal{J}_+ \mathcal{J}_-) \lesssim \\ &\lesssim \left[g^{\mu\nu} - \frac{p_1^\mu p_1^\nu}{M_W^2} \right] \left[g^{\mu'\nu'} - \frac{p_2^{\mu'} p_2^{\nu'}}{M_W^2} \right] \times \frac{E_W^{-4}}{E_W^2} \cdot E_W^{-2} E_W \cdot E_W^{-2} E_W. \end{aligned} \quad (\text{A.18})$$

As a result, the non-factorizable correction is shown to acquire at high energies an additional (screening) factor

$$\delta_{\text{nf}} \sim \frac{1}{E_W^4} \sim (1 - \beta)^2. \quad (\text{A.19})$$

To make the consideration complete we turn now to the axial piece of the Born decay cross-section, $\Delta_A^{\mu\nu}$ term in (A.12). It is possible to treat this contribution in an analogous way as before. Here we present the result of the integration over the decay phase-space

$$\begin{aligned} \mathcal{I}_{\text{nf}, A}^{\mu\nu\alpha} &= \int d^4 k_1 \delta(k_1^2) \delta(M_W^2 - 2(p_1 \cdot k_1)) \times \epsilon^{\mu\nu k_1 p_1} \times \left[\frac{p_1^\alpha}{k p_1} - \frac{k_1^\alpha}{k k_1} \right] = \\ &= B \left[k^\alpha \epsilon^{\mu\nu k p_1} - k^2 \epsilon^{\mu\nu \alpha p_1} \right] + C \left[p_1^\alpha \epsilon^{\mu\nu k p_1} - (k p_1) \epsilon^{\mu\nu \alpha p_1} \right], \end{aligned} \quad (\text{A.20})$$

where

$$B = -\frac{\pi}{8} \frac{M_W^4}{(k p_1)^3} \left[1 + \ln \frac{M_W^2 k^2}{4(k p_1)^2} \right], \quad C = +\frac{\pi}{8} \frac{M_W^2}{(k p_1)^2} \left[1 + \frac{3k^2 M_W^2}{2(k p_1)^2} \ln \frac{M_W^2 k^2}{4(k p_1)^2} \right]. \quad (\text{A.21})$$

Note that $B \sim E^0$ and $C \sim E^0$ at high energy (again we keep track of the energy dependence only). However, the Lorentz structure of (A.20) is different from that in the Born approximation. Non-factorizable current integrated over the angles of the decay products $\mathcal{I}_{\text{nf}, A}^{\mu\nu\alpha}$ is suppressed by one power of energy as compared to the Born approximation, $\sim E_W^{-1}$. Estimate similar to (A.18) shows that the relative non-factorizable correction behaves as $\sim E_W^{-4}$, where one half of the suppression comes from the phase-space of the photon, $\sim E_W^{-2}$, and another one comes from the interference between two non-factorizable currents, $\sim E_W^{-1}$ from each one. Thus, the result (A.19) remains valid when the axial contribution is taken into account.

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